Diffraction from HERA and Tevatron to LHC

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Workshop on physics with forward proton taggers at the Tevatron and LHC
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- results
- theory
- predictions
Topics

Soft diffraction
- Elastic and total cross sections
- $M^2$-scaling
- Soft diffraction cross sections
- Multigap diffraction

Diffractive DIS at HERA
- Derive $F_{2D3}$
- Explain flat ratio of $F_{2D3} / F_2$
- Explain rise of $\varepsilon$ (or $\alpha_{IP}$) with $Q^2$

Hard diffraction at the Tevatron
- Explain ratio of $F_{ij}(SD) / F_{ij}(ND)$ – magnitude and shape!
- Double-gap hard diffraction

Diffraction at the LHC
- Soft and hard single and multigap diffraction

Determine:
- triple-pomeron coupling
- pomeron intercept
- diffractive cross section using soft parton densities

Predict from hard plus soft parton densities
Diffraction at CDF in Run I

- Elastic scattering: PRD 50 (1994) 5518
- Total cross section: PRD 50 (1994) 5550
- Diffraction

**SOFT diffraction**

- Non-Diffractive (ND)
- Single-Diffractive (SD)
- Double Diffractive (DD)
- Double Pomeron Exchange (DPE)
- Single + Double Diffractive (SDD)

**HARD diffraction**

- Gap
- Jet
- Jet+Jet

**PRL reference**

- W 78 (1997) 2698
- JJ 74 (1995) 855
- JJ 85 (2000) 4217
- JJ 79 (1997) 2636
- JJ 80 (1998) 1156
- JJ 81 (1998) 5278
- b-quark 84 (2000) 232
- J/ψ 87 (2001) 241802

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Two (most) important results

**Soft Diffraction**

\[ \frac{d\sigma}{dM^2} \propto \frac{1}{(M^2)^{1+\epsilon}} \]

**Hard Diffraction**

\[ R_{jj}(x) = \frac{F_{jj}^{SD}(x)}{F_{jj}^{ND}(x)} \propto x^{-0.45} \]

\[ \langle \xi \rangle = 0.04 \quad 0.05 \quad 0.06 \quad 0.07 \quad 0.08 \quad 0.09 \]

\[ \Delta_x = 0.01 \]

\[ E_T^{\text{jet1,2}} > 7 \text{ GeV} \]

\[ |t| < 1.0 \text{ GeV}^2 \]

\[ \beta = 0.5 \]
Soft Double Pomeron Exchange

- Roman Pot triggered events
- $0.035 < \xi - \bar{p} < 0.095$
  $|t - \bar{p}| < 1 \text{ GeV}^2$
- $\xi$-proton measured using

$$\xi_p = \frac{1}{\sqrt{s}} \sum_{\text{all particles}} E_T^i \cdot e^{\eta_i}$$

- Data compared to MC based on Pomeron exchange with
- Pomeron intercept $\varepsilon = 0.1$

- Good agreement over 4 orders of magnitude!
Total Cross Section

- $\sigma_t$ exhibits universal rise with energy
- the falling term at low energies has NOTHING to do with this rise!
- **POWER LAW** behavior:
  \[
  \sigma_t = \beta^2_{IP-p}(0) \cdot s^\epsilon = \sigma_o e^{\epsilon \ln s} = \sigma_o e^{\epsilon \Delta y}
  \]
  t=0 elastic scattering amplitude

Parton model: # of wee partons grows exponentially

Im $f_{el}(\Delta y, t) \propto e^{(\epsilon + \alpha_t)\Delta y}$
Single Diffraction Variables

- **SOFT DIFFRACTION**
  \[ \Delta \eta = -\ln \xi \]
  \[ \xi = \frac{\Delta P_L}{P_L} \text{ fractional momentum loss of scattered hadron} \]
  Variables: \((\xi, t)\) or \((\Delta \eta, t)\)

- **HARD DIFFRACTION**
  \[ x_{Bj} = \sum E^\text{jet}_T e^{-\eta^\text{jet}} / \sqrt{s} \]
  \[ x = \beta \xi \leq \xi \]
  Additional variables: \((x, Q^2)\)
Soft Single Diffraction Phenomenology

Factorization & (re)normalization

\[ \Delta y = \ln s - \Delta y' \]

\[ \frac{d^2 \sigma}{d\Delta y' \, dt} = f_{IP/p}(\Delta y, t) \times \sigma_{IP-p}(\Delta y') \]

\[ C \cdot F_p^2(t) \cdot \left( e^{[\varepsilon + \alpha' \cdot \Delta y]} \right)^2 \times \kappa \times \sigma_o e^{\varepsilon \Delta y'} \]

\[ \kappa = \frac{g_{IP-IP-IP}(t)}{\beta_{IP-p}(0)} \]

Gap probability: Normalize to unity

KG, PLB 358 (1995) 379

COLOR FACTOR
The factors $\kappa$ and $\epsilon$

Experimentally: $$\kappa = \frac{g_{\text{IP}-\text{IP}}}{\beta_{\text{IP}-p}} = 0.17 \pm 0.02 \quad \leftarrow \text{KG&JM, PRD 59 (114017) 1999}$$

Theoretically: $$\kappa = f_g \times \frac{1}{N_c^2 - 1} + f_q \times \frac{1}{N_c} \xrightarrow{Q^2 \to 0} \approx 0.75 \times \frac{1}{8} + 0.25 \times \frac{1}{3} = 0.18$$

$$x \cdot f(x) = \frac{1}{x^\lambda}$$

$$\lambda_g = 0.20$$
$$\lambda_q = 0.04$$
$$\lambda_R = -0.5$$

$$\epsilon = \lambda_g \cdot w_g + \lambda_q \cdot w_q = 0.12$$
Soft Single Diffraction Data

\[ p(\bar{p}) + p \rightarrow p(\bar{p}) + X \]

**Regge**

\[ \sigma \sim s^{2\epsilon} \]

**Total cross section**

KG, PLB 358 (1995) 379

**Differential cross section**

KG&JM, PRD 59 (114017) 1999

- Differential shape agrees with Regge
- Normalization is suppressed by factor \( \propto s^{2\epsilon} \)
- Renormalize Pomeron flux factor to unity

**Soft Single Diffraction Data**

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**M^2 SCALING**
Central and Double Gaps

- **Double Diffraction Dissociation**
  - One central gap

- **Double Pomeron Exchange**
  - Two forward gaps

- **SDD: Single+Double Diffraction**
  - Forward + central gaps
Two-Gap Diffraction (hep-ph/0205141)

\[
\frac{d^5\sigma}{\prod_{i=1-5} dV_i} = C \times F_p^2(t_1) \prod_{i=1-2} \left\{ e^{(\epsilon+\alpha'_i t_i)\Delta y_i} \right\}^2 \times \kappa^2 \left\{ \sigma_o e^{\epsilon(\Delta y'_1+\Delta y'_2)} \right\}
\]

Integral \( \sim s^{2\epsilon} \) \( \sim e^{2\epsilon \Delta y} \)

Renormalization removes the s-dependence \( \rightarrow \) SCALING

5 independent variables

1. \( y'_1 \)
2. \( y_2 \)
3. \( t_1 \)
4. \( t_2 \)
5. \( \Delta y = \Delta y_1 + \Delta y_2 \)

Color factor

Integral

\[
\Delta y_1 \quad \Delta y'_1 \quad \Delta y_2 \quad \Delta y'_2
\]

Gap probability

Sub-energy cross section (for regions with particles)

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Central and Double-Gap CDF Results

Differential shapes agree with Regge predictions

One-gap cross sections require renormalization

Two-gap/one-gap ratios are \( \approx \kappa = 0.17 \)
Soft Diffraction Summary

**Multigap variables**
- $\Delta y_i$ — rapidity gap regions
- $\kappa$ — color factor = 0.17
- $\Delta y'_j$ — particle cluster regions
- also:
  - $t_i$ — $t$-across gap
  - $\eta^o_{i,j}$ — centers of floating gap/clusters

**Parton model amplitude**

$$f(\Delta y, t) \propto e^{(\varepsilon + \alpha' t) \Delta y}$$

**Differential cross section**

$$\frac{d^{\text{var}} \sigma}{\prod_{i-\text{var}} dV_i} = C \times F_p^2(t_1) \prod_{\text{i-gaps}} \left\{ e^{(\varepsilon + \alpha' t_i) \Delta y_i} \right\}^2 \times \kappa^n \left\{ \sigma_o e^{\varepsilon \Sigma_j \Delta y'_j} \right\}$$

form factor for surviving nucleon

**Normalized gap probability**

Sub-energy cross section

color factor: one $\kappa$ for each gap

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\[ F_2(x, Q^2) = c \ x^{-\lambda} \]

[from the talk of E. Tassi @ Small-x and Diffraction 2003, Fermilab]

\[ \lambda(Q^2) \text{ versus } Q^2 \]

\[ F_2 \text{ from Compton analysis (H1)} \]

**\( F_2 = c x^{-\lambda}, \ x < 0.01 \)**

- ZEUS slope fit 2001 prel.
- H1 svtx00 prel. + ZEUS BPT
- H1 svtx00 prel. + NMC
- H1 96/97 + H1 svtx00 prel.
- H1 96/97

**\( \lambda = \alpha \ln[Q^2/\Lambda^2] \)**

- Extrapolation

**\( Q^2 = 0.5 \text{ GeV}^2 \)**

**\( Q^2 = 2 \text{ GeV}^2 \)**

**\( Q^2 = 7 \text{ GeV}^2 \)**

- H1 QEDC 97 prel
- E665
- H1 SV 00 prel
- NMC
- H1 99 prel
- SLAC
- ZEUS BPT
- ALLM97
- Fractal
Diffractive DIS @ HERA

\[ x \cdot f(x) = \frac{1}{x^\varepsilon} \]

Power-law region
\[ \xi_{\text{max}} = 0.1 \]
\[ x_{\text{max}} = 0.1 \]
\[ \beta < 0.05 \xi \]

\[ F_2^{D3}(Q^2, x, \xi) \propto \frac{1}{\xi^{1+\varepsilon}} \cdot F_2(Q^2, x) \propto \frac{1}{\xi^{1+\varepsilon}} \cdot \frac{C(Q^2)}{(\beta \xi)^{\lambda(Q^2)}} \Rightarrow \frac{A}{\xi^{1+\varepsilon+\lambda}} \cdot \kappa \cdot \frac{C}{\beta^\lambda} \]

\[ R_{ND}^{SD} \equiv \frac{F_2^{D3}(Q^2, x, \xi)}{F_2(Q^2, x)} = \frac{A}{\xi^{1+\varepsilon}} \cdot \kappa^{\text{fixed} \xi} \rightarrow \text{constant} \]

\[ 2\varepsilon_{DIS}^{D} = \varepsilon + \lambda(Q^2) \]
At fixed $x_{IP}$:

$F_2^{D3}(x_{IP}, x, Q^2)$ evolves as $F_2(x, Q^2)$ independent of the value of $x$
Pomeron Intercept in DDIS

H1 Diffractive Effective $\alpha_{IP}(0)$

- $\alpha_{IP}(0)$
- $Q^2 [\text{GeV}^2]$
- Inclusive
- $1+\lambda$
- $1+(\varepsilon+\lambda)/2$

$\alpha_{IP}(0)$ vs $Q^2$ graph.

Legend:
- 1994
- 1997 (prel)
- 1999 (prel)

软 Pomeron 截距表达式:

$$1+\frac{(\varepsilon+\lambda)}{2}$$
Diffractive Dijets @ Tevatron

Test QCD factorization

\[ F_{JJ}^D(\beta) \]

suppressed at the Tevatron relative to extraplations from HERA parton densities

\[ F_{JJ}^D(\xi, \beta) = C \beta^{-n} \xi^{-m} \]

Regge factorization holds

\[ m \approx 1 \Rightarrow \text{Pomeron exchange} \]
\[ R(x) = \frac{F_{jj}^{SD}(x)}{F_{jj}^{ND}(x)} \]

\[ R(x) \Big|_{0.035<\xi<0.095} = \frac{(6.1 \times 10^{-4})}{x^{0.45}} \]
\[ R_{jj} = F_{jj}^{SD}/F_{jj}^{ND} \]

Power-law region
\[ \xi_{\text{max}} = 0.1 \quad \beta < 0.05 \xi \]

\[
F^{SD}(Q^2, x, \xi) \propto \frac{1}{\xi^{1+\varepsilon}} \cdot F^{ND}(Q^2, x) \propto \frac{1}{\xi^{1+\varepsilon}} \cdot \frac{C(Q^2)}{(\beta \xi)^{\lambda(Q^2)}} \Rightarrow A_{\text{RENORM}} \cdot \frac{\kappa \cdot C}{\xi^{1+\varepsilon+\lambda}}
\]

\[
A = \int_{\xi - \text{min}}^{\xi = 0.1} \frac{d\xi}{\xi^{1+\varepsilon+\lambda}} = (\varepsilon + \lambda) \left( \frac{M_{jj}^2}{\beta x_{\text{max}} s} \right)^{\varepsilon + \lambda}
\]

\[
R_{jj} = \frac{A}{\xi^{1-\lambda}} \cdot \frac{1}{x^{\varepsilon+\lambda}}
\]
**RENORM prediction of R(x) vs data**

- Ratio of diffractive to non-diffractive structure functions is predicted from PDF’s and color factors with no free parameters.

- \( F_{jj}(\beta,\xi) \) correctly predicted

- Test: processes sensitive to quarks will have more flat \( R(x) \) – diff \( W \)?

\[
R(x) \bigg|_{0.035 < \xi < 0.095}^{\text{DATA}} = \left(6.1 \times 10^{-4}\right) x^{0.45}
\]

\[
R(x) \bigg|_{0.035 < \xi < 0.095}^{\text{RENORM}} \approx \left(4.0 \times 10^{-4}\right) x^{0.55}
\]
HERA vs Tevatron

\[ F^D(Q^2, \beta, \xi) \xrightarrow{\text{TEVATRON}} (\varepsilon + \lambda) \left( \frac{M^2_{jj}}{\beta x_{\text{max}} s} \right)^{\varepsilon + \lambda} \frac{1}{\xi^{1+\varepsilon+\lambda}} \cdot \kappa \cdot \frac{C}{\beta \lambda} \]

(re)normalized gap probability

\[ F^D(Q^2, \beta, \xi) \xrightarrow{\text{HERA}} 0.76 \times \frac{1}{\xi^{1+\varepsilon+\lambda}} \cdot \kappa \cdot \frac{C}{\beta \lambda} \]

Pomeron flux

<table>
<thead>
<tr>
<th></th>
<th>HERA</th>
<th>Tevatron</th>
<th>Tev/HERA</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\varepsilon + \lambda)_{\text{effective}})</td>
<td>--</td>
<td>0.55</td>
<td>--</td>
</tr>
<tr>
<td>Normalization</td>
<td>0.76</td>
<td>0.042</td>
<td>0.06</td>
</tr>
<tr>
<td>(R(x) = F^D(x)/F(x))</td>
<td>flat</td>
<td>(x^{-(\varepsilon + \lambda)_{\text{eff}}})</td>
<td>(\approx x^{-0.5})</td>
</tr>
<tr>
<td>(\varepsilon_{\text{eff}} = [\varepsilon + \lambda(Q^2)]/2)</td>
<td>(~ 0.2)</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
Another issue

QCD evolution

$R_{jj}(x_{Bj}) \text{ vs } Q^2$

No appreciable $E_T^2$ dependence observed within $100 < E_T^2 < 1600 \text{ GeV}^2$
Dijets in Double Pomeron Exchange

Test of factorization

\[ R_{SD/ND} \]
\[ R_{DPE/SD} \]

\( R_{SD} \approx 5 \times R_{ND} \)

The second gap is less suppressed!!!

Factorization breaks down,

**but** see next slide!
The diffractive structure function derived from double-gap events approximately agrees with expectations from HERA.
SUMMARY

Soft and hard conclusions

Soft Diffraction

Hard Diffraction

{ Use reduced energy cross section

Pay a color factor $\kappa$ for each gap

Get gap size from renormalized $P_{\text{gap}}$

Diffraction is an interaction between low-x partons subject to color constraints
Inclusive Diffractive Higgs at the LHC

\[ p+p \rightarrow p\text{-gap-(H+X)-gap-p} \]

\[ \ln s'_{LHC} \approx \ln s_{Tevatron} \]

\[ \sigma^D(LHC) \sim \kappa^2 \ast \sigma^{ND} (Tevatron) \]

\[ \Rightarrow (0.17)^2 \ast 1 \text{ pb} = 30 \text{ fb} \]