Diffractive Cross Sections Implemented in PYTHIA8-MBR vs LHS Results

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http://www.weizmann.ac.il/conferences/lowX/
Total pp cross section: predicted in a unitarized parton model approach, which does not employ eikonalization and does not depend on the $\rho$-value.

Diffractive cross sections:
- SD - single dissociation: one of the protons dissociates.
- DD - double dissociation: both protons dissociate.
- CD – central diffraction: neither proton dissociates, but there is central diffractive production of particles.

Triple-Pomeron coupling: uniquely determined.


This is an updated version of a talk presented in DIS-2013.
DIFFRACTION IN QCD

Non-diffractive events

- color-exchange $\Rightarrow$ $\eta$-gaps exponentially suppressed

Diffractive events

- Colorless vacuum exchange
- $\eta$-gaps not suppressed

Goal: probe the QCD nature of the diffractive exchange
DEFINITIONS

SINGLE DIFFRACTION

\[ 1 - x_L \equiv \xi = \frac{M_X^2}{s} \]
Forward momentum loss

\[ \xi_{\text{CAL}} = \sum_{i=1}^{\text{all}} E_{i-\text{tower}} T e^{-\eta_i} \]

since no radiation \( \Rightarrow \) no price paid for increasing diffractive-gap width

\[ \left( \frac{d\sigma}{d \Delta \eta} \right)_{t=0} \approx \text{constant} \Rightarrow \frac{d\sigma}{d\xi} \propto \frac{1}{\xi} \Rightarrow \frac{d\sigma}{dM^2} \propto \frac{1}{M^2} \]
DIFFRACTION AT CDF

Elastic scattering

$\sigma_T = \text{Im} f_{el}(t=0)$

Total cross section

OPTICAL THEOREM

SD

Single Diffraction or Single Dissociation

SD + DD

Single + Double Diffraction (SDD)

DD

Double Diffraction or Double Dissociation

DPE/CD

Double Pom. Exchange or Central Dissociation

exclusive $JJ...ee...\mu\mu...\gamma$

$\bar{p}$

$p$

Lowx13, Israel

Diffractive x-sections in PYTHIA8-MBR vs LHC

K. Goulianos
### Basic and combined diffractive processes

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Basic Diffractive Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD$_\bar{p}$</td>
<td>$\bar{p}p \rightarrow \bar{p} + \text{gap} + [p \rightarrow X_p]$,</td>
</tr>
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<td>SD$_p$</td>
<td>$\bar{p}p \rightarrow [\bar{p} \rightarrow X_{\bar{p}}] + \text{gap} + p$,</td>
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<tr>
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**Diffractive x-sections in PYTHIA8-MBR vs LHC**

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Regge theory – values of $s_0$ & $g_{PPP}$?

\[ \alpha(t) = \alpha(0) + \alpha't \quad \alpha(0) = 1 + \varepsilon \]

\[ \sigma_T = \beta_1(0) \beta_2(0) \left( \frac{s}{s_0} \right)^{\alpha(0)-1} = \sigma_T^{p\bar{p}} \left( \frac{s}{s_0} \right)^{e} \]  

\[ \frac{d\sigma_{el}}{dt} = \frac{\beta_1^2(t) \beta_2^2(t)}{16\pi} \left( \frac{s}{s_0} \right)^{2[\alpha(t)-1]} = \frac{\sigma_T^{p\bar{p}}}{16\pi} e^{b_{el}(s)t} \]  

\[ F^4(t) \approx e^{b_{el}t} \Rightarrow b_{el}(s) = b_{0,el} + 2\alpha' \ln \left( \frac{s}{s_0} \right) \]  

\[ \frac{d^2\sigma_{sd}}{dt d\xi} = \frac{\beta_1^2(t)}{16\pi} \xi^{1-2\alpha(t)} \left[ \beta_2(0) g(t) \left( \frac{s'}{s_0'} \right)^{\alpha(0)-1} \right] \]

\[ = f_{p/p}(\xi, t) \sigma_T^{p\bar{p}}(s', t) \]  

Parameters:
- $s_0, s'_0$ and $g(t)$
- set $s'_0 = s_0$ (universal IP)
- determine $s_0$ and $g_{PPP}$ – how?
A complication ... → Unitarity!

\[
\left( \frac{d\sigma_{el}}{dt} \right)_{t=0} \sim \left( \frac{s}{s_0} \right)^{2\epsilon}, \quad \sigma_t \sim \left( \frac{s}{s_0} \right)^{\epsilon}, \quad \text{and} \quad \sigma_{sd} \sim \left( \frac{s}{s_0} \right)^{2\epsilon}
\]

- \( \sigma_{sd} \) grows faster than \( \sigma_t \) as \( s \) increases *
  → unitarity violation at high \( s \)
  (similarly for partial x-sections in impact parameter space)

- the unitarity limit is already reached at \( \sqrt{s} \sim 2 \text{ TeV} \)!

- need unitarization
Factor of ~8 (~5) suppression at √s = 1800 (540) GeV

diffractive x-section suppressed relative to Regge prediction as √s increases.

see KG, PLB 358, 379 (1995)

Interpret flux as gap formation probability that saturates when it reaches unity.

Renormalization breaking in soft diffraction.

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Single diffraction renormalized - 1


2 independent variables: \( t, \Delta y \)

\[
\frac{d^2 \sigma}{dt \, d\Delta y} = C \cdot F_p^2 (t) \cdot \left\{ e^{(\varepsilon + \alpha' t) \Delta y} \right\}^2 \cdot \kappa \cdot \left\{ \sigma_o e^{\varepsilon \Delta y'} \right\}
\]

color factor

\[
\kappa = \frac{g_{IP-IP-IP}(t)}{\beta_{IP-p-p}(0)} \approx 0.17
\]

gap probability

Gap probability ➔ (re)normalize to unity

Lowx13, Israel     Diffractive x-sections in PYTHIA8-MBR vs LHC     K. Goulianos
Single diffraction renormalized - 2

\[ \kappa = \frac{g_{\text{IP-IP-IP}}(t)}{\beta_{\text{IP-p-p}}(0)} \approx 0.17 \]

Experimentally:
\[ \kappa = \frac{g_{\text{IP-IP-IP}}}{\beta_{\text{IP-p}}} = 0.17 \pm 0.02, \quad \varepsilon = 0.104 \]

QCD:
\[ \kappa = f_g \times \frac{1}{N_c^2 - 1} + f_q \times \frac{1}{N_c} \quad Q^2 = 1 \rightarrow \approx 0.75 \times \frac{1}{8} + 0.25 \times \frac{1}{3} = 0.18 \]
Single diffraction renormalized - 3

\[
\frac{d^2 \sigma_{sd}(s, M^2, t)}{dM^2 dt} = \left[ \frac{\sigma_0}{16\pi} \sigma_0^{IPp} \right] \frac{s^{2\epsilon}}{N(s, s_o)} \frac{e^{bt}}{(M^2)^{1+\epsilon}}
\]

\[
b = b_0 + 2\alpha' \ln \frac{s}{M^2} \quad s_o^{CMG} = (3.7 \pm 1.5) \text{ GeV}^2
\]

\[
N(s, s_o) \equiv \int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi \int_{t=0}^{-\infty} dt f_{IP/p}(\xi, t) \xrightarrow{s \to \infty} \sim s_o^{\epsilon} \frac{s^{2\epsilon}}{\ln s}
\]

\[
\lim_{s \to \infty} \frac{d^2 \sigma_{sd}(s, M^2, t)}{dM^2 dt} \sim \ln s \frac{e^{bt}}{(M^2)^{1+\epsilon}}
\]

\[
\sigma_{sd} \xrightarrow{s \to \infty} \sim \frac{\ln s}{b \to \ln s} \Rightarrow \text{const}
\]

set to unity \( \Rightarrow \) determines \( s_o \)
$M^2$ distribution: data

$\frac{d\sigma}{dM^2} \propto \frac{s^{2\varepsilon}}{(M^2)^{1+\varepsilon}} \rightarrow 1$

Independent of $s$ over 6 orders of magnitude in $M^2$

$\Rightarrow M^2$ scaling

$\Rightarrow$ factorization breaks down to ensure $M^2$ scaling!
Scale $s_0$ and $PPP$ coupling

Pomeron flux: interpret as gap probability
- set to unity: determines $g_{PPP}$ and $s_0$

\[
\frac{d^2\sigma_{SD}}{dt\,d\xi} = f_{IP/p}(t, \xi) \sigma_{IP/p}(s\xi) \quad \text{with} \quad s_0^{-\varepsilon/2} \cdot g_{PPP}(t)
\]

Pomeron-proton x-section

- Two free parameters: $s_0$ and $g_{PPP}$
- Obtain product $g_{PPP} \cdot s_0^{-\varepsilon/2}$ from $\sigma_{SD}$
- Renormalized Pomeron flux determines $s_0$
- Get unique solution for $g_{PPP}$

KG, PLB 358 (1995) 379
Saturation at low $Q^2$ and small-$x$

**Figure from a talk by Edmond Iancu**

\[ Y = \ln \frac{1}{x} \]

- Saturation: $\ln Q_s^2(Y) = \lambda Y$
- Dilute system
- BFKL
- DGLAP

**Total Single Diffractive Cross Section (mb)**

- $\xi < 0.05$
- Albrow et al.
- Armitage et al.
- UA4
- CDF
- E710
- Cool et al.

**Graph:**

- $\sqrt{s}$ (GeV)
- $\ln \Lambda_{QCD}^2$
- $\ln Q^2$

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Diffractive x-sections in PYTHIA8-MBR vs LHC

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DD at CDF

\[
\frac{d^3\sigma_{\text{DD}}}{dtdM_1^2dM_2^2} = \frac{d^2\sigma_{\text{SD}}}{dtdM_1^2} \frac{d^2\sigma_{\text{SD}}}{dtdM_2^2} \left/ \frac{d\sigma_{\text{el}}}{dt} \right. \\
= \frac{[\kappa \beta_1(0)/\beta_2(0)]^2}{16\pi} \frac{s^{2\epsilon} e^{b_{\text{DD}t}}}{(M_1^2 M_2^2)^{1+2\epsilon}}
\]

\[
\frac{d^3\sigma_{\text{DD}}}{dtd\Delta\eta d\eta_c} = \left[ \frac{\kappa \beta^2(0)}{16\pi} e^{2[\alpha(t) - 1]\Delta\eta} \right] \left[ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right)^\epsilon \right]
\]

gap probability  
x-section

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Diffractive x-sections in PYTHIA8-MBR vs LHC  
K. Goulianos
SDD at CDF

- Excellent agreement between data and MBR (MinBiasRockefeller) MC

\[
\frac{d^5 \sigma}{d t_p dt d\xi_p d\Delta \eta d\eta_c} = \left[ \frac{\beta(t)}{4\sqrt{\pi}} e^{[\alpha(t_p)-1]\ln(1/\xi)} \right]^2 \times \kappa \left[ \frac{\beta(0)}{4\sqrt{\pi}} e^{[\alpha(t)-1]\Delta \eta} \right]^2 \kappa \left[ \beta^2(0) \left( \frac{s''}{s_o} \right)^\epsilon \right]
\]
Excellent agreement between data and MBR

- low and high masses are correctly implemented
Diffractive x-sections

\[
\frac{d^2 \sigma_{SD}}{d t d \Delta y} = \frac{1}{N_{gap}(s)} \left[ \frac{\beta^2(t)}{16\pi} e^{2[\alpha(t)-1] \Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right) \epsilon \right\},
\]

\[
\frac{d^3 \sigma_{DD}}{d t d \Delta y dy_0} = \frac{1}{N_{gap}(s)} \left[ \frac{\kappa \beta^2(0)}{16\pi} e^{2[\alpha(t)-1] \Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right) \epsilon \right\},
\]

\[
\frac{d^4 \sigma_{DPE}}{d t_1 d t_2 d \Delta y dy c} = \frac{1}{N_{gap}(s)} \left[ \prod_i \left[ \frac{\beta^2(t_i)}{16\pi} e^{2[\alpha(t_i)-1] \Delta y_i} \right] \right] \cdot \kappa \left\{ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right) \epsilon \right\}
\]

\[
\beta^2(t) = \beta^2(0) F^2(t)
\]

\[
F^2(t) = \left[ \frac{4m_p^2 - 2.8t}{4m_p^2 - t} \left( \frac{1}{1 - \frac{t}{0.71}} \right)^2 \right]^2 \approx a_1 e^{b_1 t} + a_2 e^{b_2 t}
\]

\[\alpha_1 = 0.9, \alpha_2 = 0.1, b_1 = 4.6 \text{ GeV}^{-2}, b_2 = 0.6 \text{ GeV}^{-2}, s' = s e^{-\Delta y}, \kappa = 0.17, \kappa \beta^2(0) = \sigma_0, s_0 = 1 \text{ GeV}^2, \sigma_0 = 2.82 \text{ mb or } 7.25 \text{ GeV}^{-2}\]
Total, elastic, and inelastic x-sections

\[ \sigma_{\text{ND}} = (\sigma_{\text{tot}} - \sigma_{\text{el}}) - (2\sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}}) \]

\[ \sigma_{\text{tot}}^{p\pm p} = \begin{cases} 
16.79 s^{0.104} + 60.81 s^{-0.32} & \text{for } \sqrt{s} < 1.8 \\
\sigma_{\text{tot}}^{\text{CDF}} + \frac{\pi}{s_0} \left[ \left( \ln \frac{s}{s_F} \right)^2 - \left( \ln \frac{s_{\text{CDF}}}{s_F} \right)^2 \right] & \text{for } \sqrt{s} \geq 1.8 
\end{cases} \]

\[ \sqrt{s_{\text{CDF}}} = 1.8 \text{ TeV}, \ \sigma_{\text{tot}}^{\text{CDF}} = 80.03 \pm 2.24 \text{ mb} \]
\[ \sqrt{s_F} = 22 \text{ GeV}, \ \ s_0 = 3.7 \pm 1.5 \text{ GeV}^2 \]

\[ \sigma_{\text{el}}^{p\pm p} = \sigma_{\text{tot}} \times \left( \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \right), \text{ with } \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \text{ from CMG} \]

Small extrapolation from 1.8 to 7 and up to 50 TeV


Use the Froissart formula as a saturated cross section:

\[ \sigma_t(s > s_F) = \sigma_t(s_F) + \frac{\pi}{m^2} \cdot \ln^2 \frac{s}{s_F} \]

This formula should be valid above the knee in \( \sigma_{sd} \) vs. \( \sqrt{s} \) at \( \sqrt{s}_F = 22 \text{ GeV} \) (Fig. 1) and therefore valid at \( \sqrt{s} = 1800 \text{ GeV} \).

Use \( m^2 = s_0 \) in the Froissart formula multiplied by \( 1/0.389 \) to convert it to \( \text{mb}^{-1} \).

Note that contributions from Reggeon exchanges at \( \sqrt{s} = 1800 \text{ GeV} \) are negligible, as can be verified from the global fit of Ref. [7].

Obtain the total cross section at the LHC:

\[ \sigma_{t_{LHC}} = \sigma_{t_{CDF}} + \frac{\pi}{s_0} \cdot \left( \ln^2 \frac{s_{LHC}}{s_F} - \ln^2 \frac{s_{CDF}}{s_F} \right) \]

98 ± 8 mb at 7 TeV
109 ±12 mb at 14 TeV

Main error from \( s_0 \)
Reduce the uncertainty in $s_0$

Saturation glueball?

- Giant glue-ball with $f_0(980)$ and $f_0(1500)$ superimposed, interfering destructively and manifesting as dips (???)
- glue-ball-like object $\Rightarrow$ “superball”
- mass $\rightarrow 1.9$ GeV $\Rightarrow m_s^2 = 3.7$ GeV
- agrees with RENORM $s_0 = 3.7$
- Error in $s_0$ can be reduced by factor $\sim 4$ from a fit to these data! $\Rightarrow$ reduces error in $\sigma_t$. 

Figure 8: $M_{\pi^+\pi^-}$ spectrum in DIPE at the ISR (Axial Field Spectrometer, R807 [97, 98]). Figure from Ref. [98]. See M.G.Albrow, T.D. Goughlin, J.R. Forshaw, hep-ph>arXiv:1006.1289
TOTEM vs PYTHIA8-MBR

\[ \sigma_{_{\text{inrl}}}(7 \text{ TeV}) = 72.9 \pm 1.5 \text{ mb} \]

\[ \sigma_{_{\text{inrl}}}(8 \text{ TeV}) = 74.7 \pm 1.7 \text{ mb} \]

Renormalized:

\[ \sigma_{_{\text{inrl}}}(7 \text{ TeV}) = 71.1 \pm 1.2 \text{ mb} \]

\[ \sigma_{_{\text{inrl}}}(8 \text{ TeV}) = 72.3 \pm 1.2 \text{ mb} \]

G. Latino talk at MPI@LHC, CERN 2012
CMS SD and DD x-sections vs ALICE: measurements and theory models

KG*: after extrapolation into low $\xi$ from measured CMS data using the MBR model: find details on data in Benoit Roland’s talk on Sunday at 09:05.
CMS SD and DD $x$-sections vs ALICE: measurements and theory models

KG*: after extrapolation into low $\xi$ from measured CMS data using the MBR model:
find details on data in Benoit Roland’s talk on Sunday at 09:05.

Includes ND background
Total-Inelastic Cross Sections vs model predictions

CMS pp $\sqrt{s} = 7$ TeV

- CMS - HF based
- CMS - Vtx based
- ATLAS
- TOTEM
- ALICE
- PYTHIA 6
- PYTHIA 8
- PYTHIA 8 + MBR
- PHOJET
- EPOS 1.99
- QGSJET 01
- QGSJET II-03
- QGSJET II-04
- SIBYLL 2.1
MONTE CARLO STRATEGY

- $\sigma_{\text{tot}} \rightarrow$ from SUPERBALL model
- optical theorem $\rightarrow$ $\text{Im} \ f_{\text{el}}(t=0)$
- dispersion relations $\rightarrow$ $\text{Re} \ f_{\text{el}}(t=0)$
- $\sigma_{\text{el}} \leftarrow$ using global fit
- $\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$
- differential $\sigma_{\text{sd}} \rightarrow$ from RENORM
- use *nesting* of final states for $pp$ collisions at the $P$-$p$ sub-energy $\sqrt{s'}$

*Strategy similar to that of MBR used in CDF based on multiplicities from: K. Goulianos, Phys. Lett. B 193 (1987) 151 pp*

“A new statistical description of hardonic and $e^+e^-$ multiplicity distributions“
Monte Carlo algorithm - nesting

Profile of a $pp$ inelastic collision

- $\Delta y' < \Delta y'_{\text{min}}$
-Hadronize
- Generate central gap
- Repeat until $\Delta y' < \Delta y'_{\text{min}}$
- Evolve every cluster similarly

Final state of MC w/no-gaps
SUMMARY

- Introduction

- Diffractive cross sections:
  - basic: SD1, SD2, DD, CD (DPE)
  - combined: multigap x-sections
  - ND → no diffractive gaps:
    - this is the only final state to be tuned

- Total, elastic, and total inelastic cross sections

- Monte Carlo strategy for the LHC – “nesting”

Thank you for your attention