Diffractive, Elastic, and Total pp Cross Sections at the LHC

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Contents

- ❑ References
- ❑ Single diffraction
  - ➔ Renormalization
  - ➔ triple-Pomeron coupling
  - ➔ ratio of α'/ε
- ❑ Total cross section
  - ➔ the SUPERBALL cross section
  - ➔ cross section from a global fit
- ❑ The elastic cross section
- ❑ Discussion

References

http://physics.rockefeller.edu/dino/my.html

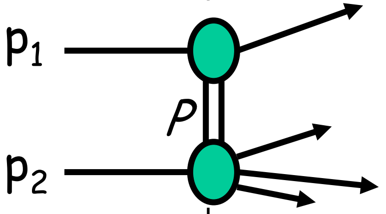
CDF	PRD 50, 5518 (1994)	σ^{el} @ 1800 & 546 GeV
CDF	PRD 50, 5535 (1994)	σ^{D} @ 1800 & 546 GeV
CDF	PRD 50, 5550 (1994)	σ^{T} @ 1800 & 546 GeV
KG-PR	Physics Reports 101, No.3 (1983) 169-219	
KG-95	PLB 358, 379 (1995)	Renormalization
	Erratum: PLB 363, 268 (1995)	
CMG-96	PLB 389, 176 (1996)	Global fit to $p^{\pm}p$, $\pi^{\pm}$, $K^{\pm}p$

Single diffraction

KG-95

Pomeron flux

s_o^{ε}



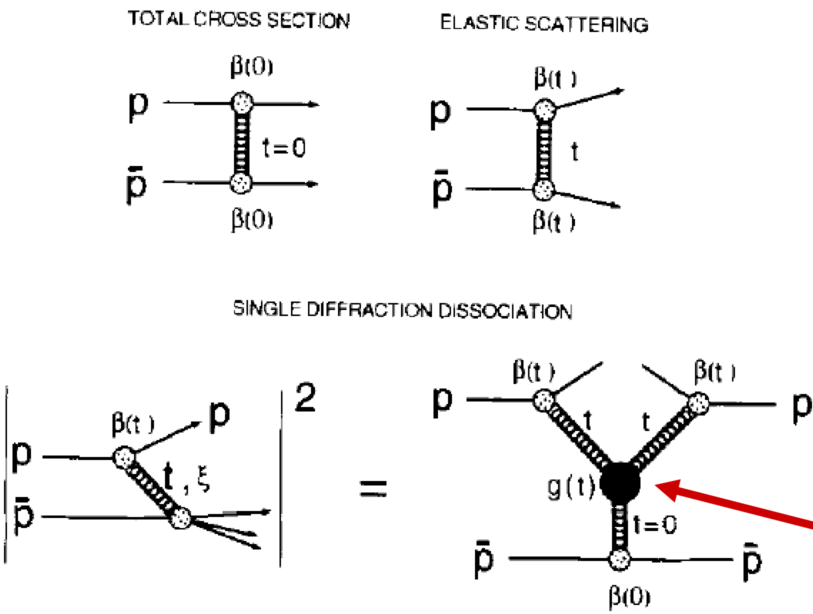
$$\frac{d^2\sigma_{SD}}{dt\,d\xi} = f_{P/p_1}(t,\xi)\cdot\sigma_{Pp}(s\xi)$$

$s_o^{-\varepsilon/2}\cdot g_{PPP}(t)$

Pomeron-proton x-section

- Two free parameters: s_o and g_{PPP}
- Obtain product $g_{PPP}\bullet s_o^{\varepsilon/2}$ from σ_{SD}
- Renormalized Pomeron flux determines s_o
- Get unique solution for g_{PPP}

Standard Regge theory



KG-95

$$\sigma_T = \beta_1(0)\beta_2(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1} = \sigma_0^{p\bar{p}}\left(\frac{s}{s_0}\right)^{\epsilon} \quad (1)$$

$$\frac{d\sigma_{el}}{dt} = \frac{\beta_1^2(t)\beta_2^2(t)}{16\pi}\left(\frac{s}{s_0}\right)^{2[\alpha(t)-1]}$$

$$= \frac{\sigma_T^2}{16\pi}\left(\frac{s}{s_0}\right)^{2\alpha' t} F^4(t) \approx \frac{\sigma_T^2}{16\pi} e^{b_{el}(s)t} \quad (2)$$

$$F^4(t) \approx e^{b_{0,el}t} \quad \Rightarrow \quad b_{el}(s) = b_{0,el} + 2\alpha' \ln\left(\frac{s}{s_0}\right) \quad (3)$$

$$\frac{d^2\sigma_{sd}}{dt\,d\xi} = \frac{\beta_1^2(t)}{16\pi}\xi^{1-2\alpha(t)}\left[\beta_2(0)\,g(t)\left(\frac{s'}{s'_0}\right)^{\alpha(0)-1}\right]$$

$$= f_{\mathcal{P}/p}(\xi,t)\,\sigma_T^{\mathcal{P}\bar{p}}(s',t) \quad (4)$$

Parameters:

- ❑ s_o, s_o' and g(t)
- ❑ set $s_o' = s_o$ (universal) *P*
- ❑ g(t) ➔ g(0) ≡ g_{PPP} see KG-PR
- ❑ determine s_o and g_{PPP} – how?

The triple-Pomeron coupling in QCD

$$\frac{d^2\sigma}{dt\, d\Delta y} = N_{gap} \cdot \underbrace{C \cdot F_p^2(t) \cdot \left\{ e^{(\varepsilon+\alpha' t)\Delta y} \right\}^2}_{P_{gap}(\Delta y, t)} \cdot \kappa \cdot \left\{ \sigma_o\, e^{\varepsilon\, \Delta y'} \right\}$$

$$\sigma_T = \sigma_o \cdot s^{\varepsilon}$$

Experimentally:

KG&JM, PRD 59 (114017) 1999

$$\kappa = \frac{g_{PPP}}{\beta_{Pp}} = 0.17 \pm 0.02$$

Color factor:

$$\kappa = f_g \times \frac{1}{N_c^2 - 1} + f_q \times \frac{1}{N_c} \xrightarrow{Q^2 = 1} \approx 0.75 \times \frac{1}{8} + 0.25 \times \frac{1}{3} = 0.18$$

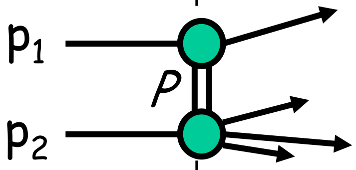
f_g: gluon fraction

f_q: quark fraction

σ^T_{sd} renormalized

KG-95

Pomeron flux

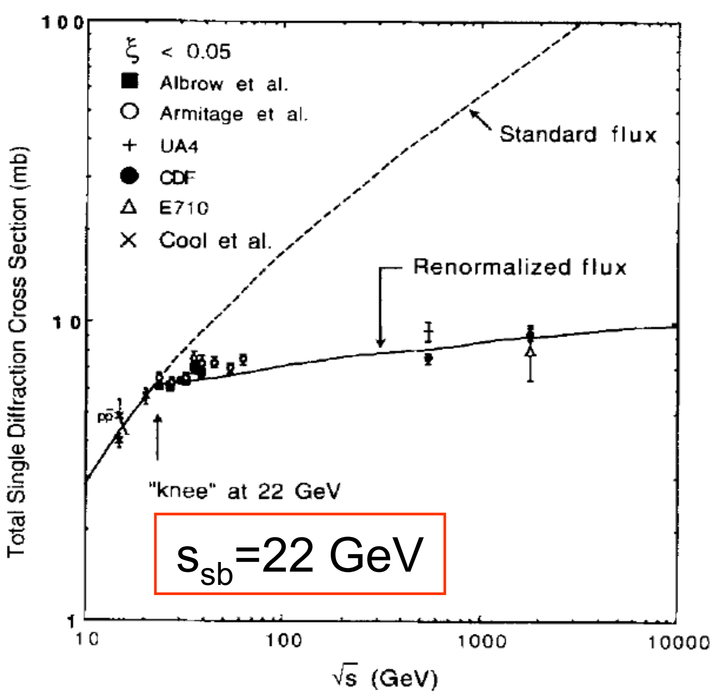


$$s_0^{\varepsilon}$$

$$\frac{d^2\sigma_{SD}}{dt\;d\xi} = f_{P/p_1}(t,\,\xi)\cdot\sigma_{Pp}(s\,\xi)$$

$$s_0^{-\varepsilon/2}\cdot g_{PPP}(t)$$

$$\int_{\xi_{min}}^{0.1}\int_{t=-\infty}^{0} f_{P/p}(t,\xi)\,d\xi\,dt \Rightarrow 1$$



s_{sb}=22 GeV

Renormalization

- $$\int_{\xi_{min}\approx 1/s}^{\zeta}\int_{t=-\infty}^{0} f_{P/p}(t,\xi)\,d\xi\,dt \approx C\cdot s^{2\varepsilon}\cdot s_0^{\varepsilon} \Rightarrow 1$$
- Flux integral depends on s and s_0
- "knee" √s-position determines the s_{sb} value where flux becomes unity ➔ get s_o
- $\delta s_o/s_o = -2\,\delta s/s = -4\,(\delta\sqrt{s})/\sqrt{s}$
- get error in s_o from error in √s-knee

Renormalization and Pumplin bound

$$\frac{d^2\sigma}{dt\,d\Delta y} = N_{gap} \cdot \underbrace{C \cdot F_p^2(t) \cdot \left\{ e^{(\varepsilon+\alpha' t)\Delta y} \right\}^2}_{P_{gap}(\Delta y, t)} \cdot \kappa \cdot \left\{ \sigma_o \, e^{\varepsilon\,\Delta y'} \right\}$$

$$N_{gap}^{-1}(s) = \int_{\Delta y, t} P_{gap}(\Delta y, t)\, d\Delta y\, dt \xrightarrow{s\to\infty} C' \cdot \frac{s^{2\varepsilon}}{\ln s}$$

$$\frac{d^2\sigma}{dt\,d\Delta y} = C'' \left[e^{\varepsilon(\Delta y - \ln s)} \cdot \ln s \right] e^{(b_0 + 2\alpha' \Delta y)t}$$

grows slower than s^{ε}

→ The Pumplin bound is obeyed at all impact parameters

σ^T_{sd} ($s\rightarrow\infty$) and ratio of α'/ε

arXiv:0812.4464v1[hep-ph] submitted to PLB

$$\frac{d^2\sigma(s,M^2,t)}{dM^2dt} = \left[\frac{\sigma_0^{pp}}{16\pi}\sigma_0^{\mathcal{P}p}\right]\frac{s^{2\epsilon}}{N(s)}\frac{1}{(M^2)^{1+\epsilon}}e^{bt} \stackrel{s\to\infty}{\longrightarrow} \left[2\alpha' e^{\frac{\epsilon b_0}{\alpha'}}\sigma_0^{\mathcal{P}p}\right]\underbrace{\frac{\ln s^{2\epsilon}}{(M^2)^{1+\epsilon}}e^{bt}}_{b=b_0+2\alpha'\ln\frac{s}{M^2}}$$

$$\sigma_{sd} \stackrel{s\to\infty}{\longrightarrow} \sigma_0^{\mathcal{P}p}\, e^{\frac{\epsilon}{2\alpha'}b_0}\, s^{\epsilon}\frac{\sum_{n=1}^{\infty}\frac{(\ln s^{\epsilon})^n}{n\,n!}}{\sum_{n=1}^{\infty}\frac{(\ln s^{2\epsilon})^n}{n\,n!}} = 2\sigma_0^{\mathcal{P}p}\, e^{\frac{\epsilon}{2\alpha'}b_0} \Rightarrow \sigma_0^{pp}$$

← Constant set to σ_o^{pp}

$$\sigma_0^{Pp} = \kappa\sigma_0^{pp}$$

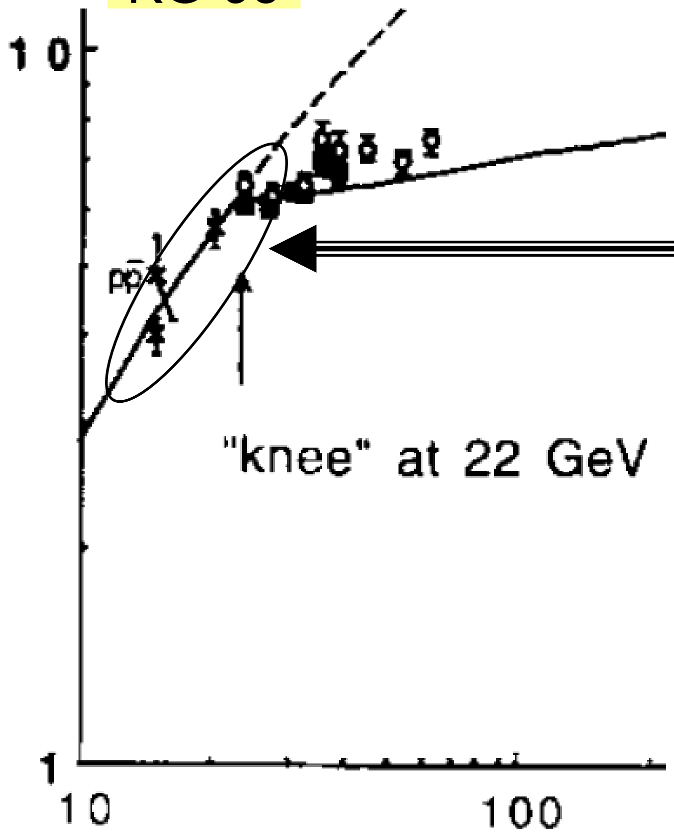
$$2\kappa\exp\left(\frac{\varepsilon b_o^{sd}}{2\alpha'}\right) = 1$$

$$b_o^{sd} = \frac{R_p^2}{2} = \frac{1}{2m_\pi^2}$$

$$\alpha' = -\varepsilon\frac{1/4m_\pi^2}{4\,\ln(2\kappa)} = 0.25\ \text{GeV}^{-2}\ (\text{using } \varepsilon = 0.08)\left[\Rightarrow \frac{\alpha'}{\varepsilon} = 3.14 \approx \pi\,!)\right]$$

The value of s_o - a bird's-eye view

KG-95

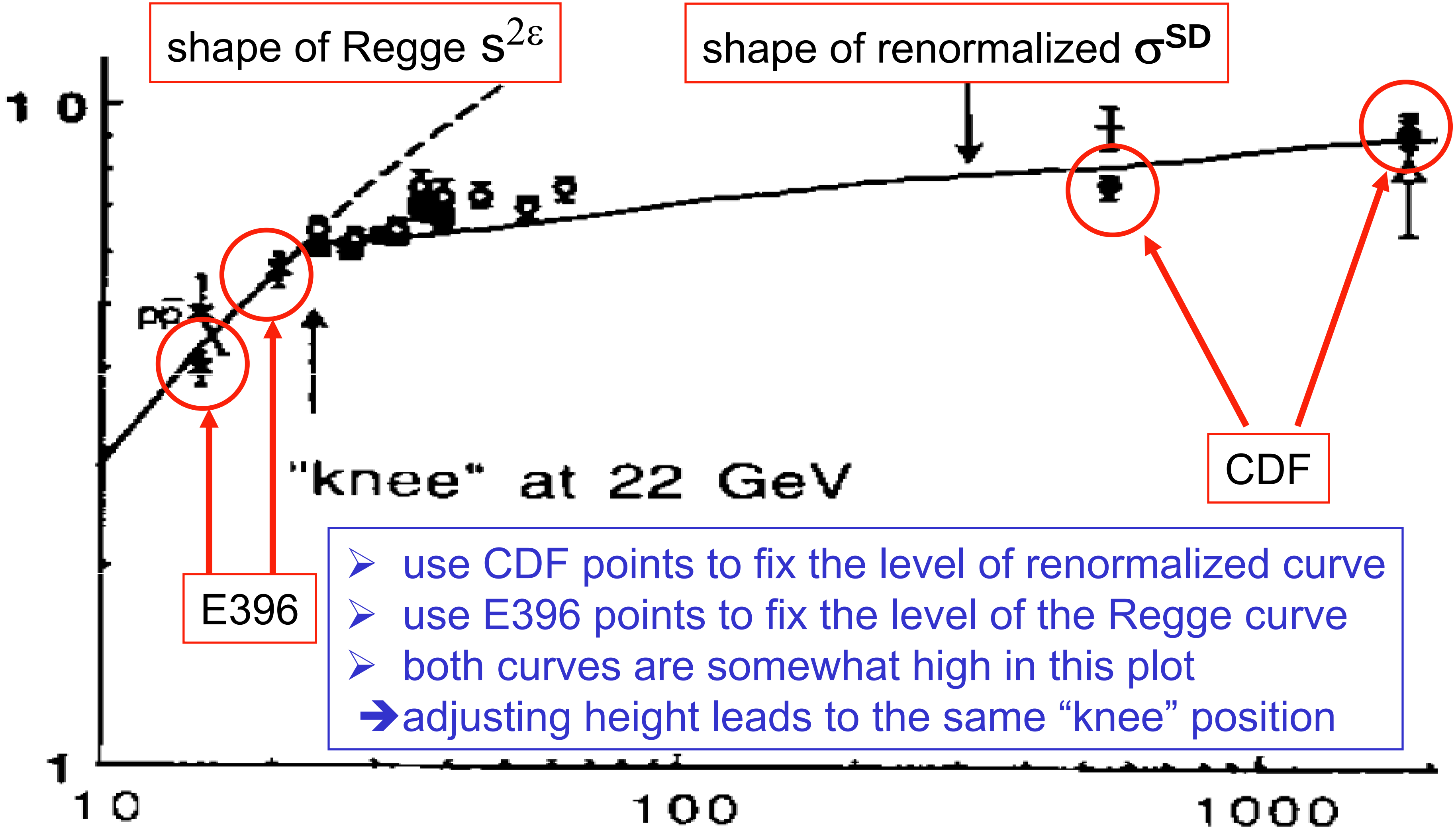


			√s (GeV)	
☒	Cool [KG]	FNAL	14	20
■	Albrow	ISR	23.3	26.9
o	Armitage	ISR	23.3	27.4

➔ error in knee position: ± 1 GeV

"knee" @ 22 ± 1 GeV ➔ s_o = 1 ± 0.2 GeV

The value of s_o - limited edition



- use CDF points to fix the level of renormalized curve
- use E396 points to fix the level of the Regge curve
- both curves are somewhat high in this plot

➔adjusting height leads to the same "knee" position

The total cross section

- Froissart-Martin bound: $\sigma \leq \frac{\pi}{m^2} \cdot \ln^2 s$ (s in GeV^2)
- For $m^2 = m_\pi^2$ ➔ $\pi / m^2 \sim 10^4$ mb – very large!
- But if $m^2 = s_o$ = (mass)^2 of a large **SUPERglueBALL**, a $\ln^2 s$ behavior *ala* Froissart-Martin an be reached at a much lower s-value, s_{sb},

$$\rightarrow \sigma(s > s_{sb}) = \sigma(s_{sb}) + \frac{\pi}{s_o} \cdot \ln^2 \frac{s}{s_{sb}}$$

- Determine s_{sb} and s_o from σ_T^{SD}
- Show that $\sqrt{s_{sb}} < 1.8$ TeV
- Show that at $\sqrt{s} = 1.8$ TeV Reggeon contributions are negligible
- Get cross section at the LHC from

$$\sigma^{LHC} = \sigma^{CDF}_{1800} + \frac{\pi}{\mathbf{s_0}} \cdot \left(\ln^2 \frac{s^{LHC}}{s_F} - \ln^2 \frac{s^{CDF}}{s_F} \right)$$

The SUPERBALL σ^T

- Froissart-Martin bound $$\sigma \leq \frac{\pi}{m^2} \cdot \ln^2 s$$
- Valid above "knee" at √s = 22 GeV and therefore at √s = 1.8 TeV
- Use superball mass

➔ $m^2 = s_o = (1 \pm 0.2)\ \text{GeV}^2$

- At √s 1.8 TeV Reggeon contributions are negligible (CMG-96))

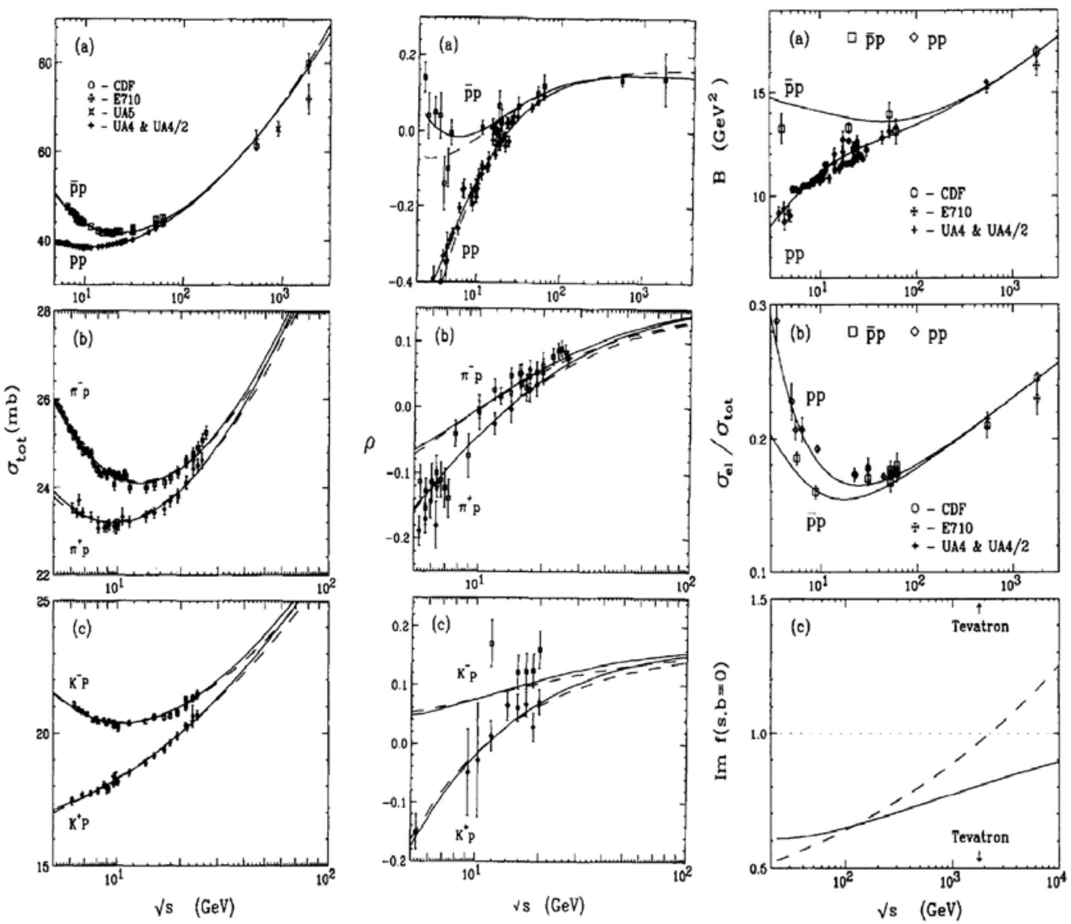
$$\sigma_{14000}^{LHC} = \sigma_{1800}^{CDF} + \frac{\pi}{s_0} \cdot \left(\ln^2 \frac{s^{LHC}}{s_F} - \ln^2 \frac{s^{CDF}}{s_F} \right) = (80.03 \pm 2.24) + (39 \pm 6) = 119 \pm 6\ \text{mb}$$

➔ compatible with CGM-96 global fit result of 114 ± 5 mb (see next 2 slides)

Global fit to p±p, π±, K±p x-sections

CMG-96 → A new determination of the soft pomeron intercept

R.J.M. Covolan[1], J. Montanha[2], K. Goulianos[3]



Use standard Regge theory

- INPUT

$$\alpha_{f/a} = 0.68 + 0.82\, t$$

$$\alpha_{\omega/\rho} = 0.46 + 0.92\, t$$

$$\alpha'_{\mathbb{P}} = 0.25\ \mathrm{GeV}^{-2}$$

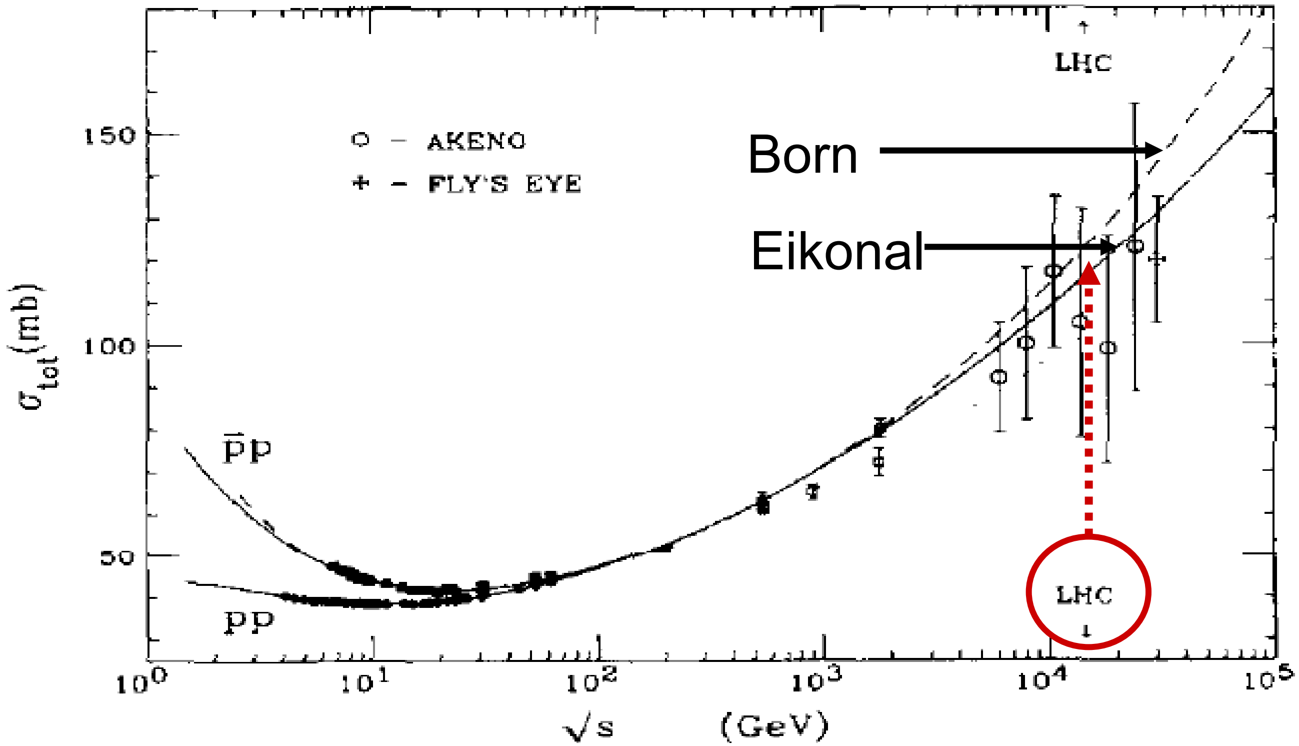
- RESULTS

$$\alpha^{Born}_{0,\mathbb{P}} = 1.104 \pm 0.002\,, \quad \alpha^{Eik}_{0,\mathbb{P}} = 1.122 \pm 0.002$$

$$\sigma^{p^{\pm}p}_{tot} = 16.79\, s^{0.104} + 60.81\, s^{-0.32} \mp 31.68\, s^{-0.54}$$

negligible

σ^T at LHC from global fit



❖ σ @ LHC √s=14 TeV: 122 ± 5 mb Born, 114 ± 5 mb eikonal
➔ error estimated from the error in ε given in CMG-96

Compare with **SUPERBALL** σ(14 TeV) = 113 ± 6 mb

caveat: s_o=1 GeV² was used in global fit!

The elastic cross section

- Optical theorem: obtain imaginary part of the amplitude from σ^T
- Dispersion relations: obtain real part of the amplitude from σ^T

σ_T

↕ **optical theorem**

$\mathrm{Im}\, f_{el}(t=0)$

↕ **dispersion relations**

$\mathrm{Re}\, f_{el}(t=0)$

- Add Coulomb amplitude
- Get differential elastic cross section and ρ-value

DISCUSSION


thank you