1 Introduction

In DIS-2015 (Spring 2015), we summarized\(^1\), the pre-LHC predictions of the total, elastic and total-inelastic, as well as the single- and double-diffractive components of the proton-proton cross section at high energies, based on the RENORM/MBR model\(^2\). We compared the measurements of the SD and DD cross sections from the Tevatron and the LHC with the predictions of the model and found excellent agreement. Good agreement was also observed between the model predictions and the total, elastic, and total inelastic cross sections obtained at the Tevatron at \(\sqrt{s} = 1.8\) TeV, and at the LHC at \(\sqrt{s} = 7\) and 8 TeV.

The success of the predictions of the RENORM/MBR model for all the above cross sections at the Tevatron and LHC up to \(\sqrt{s} = 8\) TeV prompted an extrapolation to \(\sqrt{s} = 13\) TeV, the nominal foreseen colliding-beam energy at the LHC in Summer 2015. For \(\sigma_{\text{tot}}\), \(\sigma_{\text{el}}\), and \(\sigma_{\text{inel}}\), we predicted 108 mb, 32 mb and 77 mb, respectively, with uncertainties of \(\sim 11\%\) in all cases, mainly due to the uncertainty in the energy-squared scale parameter \(s_0\) of the model.

In Summer 2015, we updated the value of \(s_0\) to a more precise one based on a tensor glueball interpretation of the Axial Field Spectrometer (AFS) exclusive charged di-pion data\(^3\)\(^4\)\(^5\). This change in RENORM/MBR decreases the uncertainties in the predictions of the total, elastic, and total-inelastic cross sections to less than 2% from Tevatron to LHC energies, with little or no effect on the mean values, and yields cross sections in excellent agreement with the measurements by ATLAS at \(\sqrt{s} = 7\) TeV and by TOTEM at \(\sqrt{s} = 7\) and 8 TeV, as discussed below.

\(^a\)Presented at Moriond QCD and High Energy Interactions, La Thuile, Italy, March 22-29, 2016.
2 RENORM cross sections

The total, elastic, and total-inelastic cross sections in the RENORM/MBR model depend on the value of the energy-squared scale parameter, \(s_0\). Quoting verbatim from Ref. 1,

"The total cross section (\(\sigma_{\text{tot}}\)) is expressed as
\[
\sigma_{\text{tot}}^{p+p} = 16.79s^{0.104} + 60.81s^{-0.32} + 31.68s^{-0.54} \quad \text{for } \sqrt{s} \leq 1.8 \text{ TeV},
\]
\[
\sigma_{\text{tot}}^{p+p} = \sigma_{\text{CDF}}^{p+p} + \frac{\pi}{s_0} \left[ (\ln \frac{s}{s_F})^2 - (\ln \frac{s_{\text{CDF}}}{s_F})^2 \right] \quad \text{for } \sqrt{s} \geq 1.8 \text{ TeV},
\]

where \(s_0\) and \(s_F\) are the energy and (Pomeron flux) saturation scales, \(s_0 = 3.7 \pm 1.5 \text{ GeV}^2\) and \(s_F = 22 \text{ GeV}\), respectively. For \(\sqrt{s} \leq 1.8 \text{ TeV}\), where there are Reggeon contributions, we use the global fit expression \(^7\), while for \(\sqrt{s} \geq 1.8 \text{ TeV}\), where Reggeon contributions are negligible, we employ the Froissart-Martin formula \(^8,9,10\). The two expressions are smoothly matched at \(\sqrt{s} \approx 1.8 \text{ TeV}\). The \(\sigma_{\text{el}}\) for \(\sqrt{s} \leq 1.8 \text{ TeV}\) is obtained from the global fit \(^7\), while for \(1.8 < \sqrt{s} \leq 50 \text{ TeV}\) we use an extrapolation of the global-fit ratio of \(\sigma_{\text{el}}/\sigma_{\text{tot}}\), which is slowly varying with \(\sqrt{s}\), multiplied by \(\sigma_{\text{tot}}\). The total non-diffractive cross section is given by
\[
\sigma_{\text{ND}} = (\sigma_{\text{tot}} - \sigma_{\text{el}}) - (2\sigma_{SD} + \sigma_{DD} + \sigma_{CD}).
\]

3 Tensor-Pomeron predictions

The partial wave analysis of the AFS exclusive \(\pi^+\pi^-\) data \(^5\), performed in terms of a fit with a model with S-wave and D-wave amplitudes as a function of the di-pion mass up to 2.3 GeV, leads to the results presented in Fig. 1.
The D-wave dominates at masses above $\sim 2$ GeV, and according to the presumed interpretation in Ref. $^5$ it corresponds to a spin-2 tensor glueball of mass $M_{tgb}$. A Gaussian fit to this enhancement yields $M_{tgb} = 2.10 \pm 0.68$ GeV. Identifying $M_{tgb}$ with the saturated glueball-like enhancement of the MBR-model parameter $s_0$ (see Eq. 2) yields $s_0 = 4.42 \pm 0.34$ GeV$^2$. Using this value in Eq. 2, we predicted for $\sigma_{tot}$, $\sigma_{el}$, and $\sigma_{inel}$ at 13 TeV cross sections of $103.7 \pm 1.9$ mb, $30.2 \pm 0.8$ mb, and $73.5 \pm 1.3$ mb, respectively. The ATLAS- and TOTEM-measured cross sections at $\sqrt{s} = 7$ and 8 TeV$^1$ are shown in Table 1 along with the MBR predictions. The measurements are in good agreement with the predictions. Also shown is a recent measurement of the total inelastic cross section by ATLAS at $\sqrt{s} = 13$ TeV$^1$, $\sigma_{inel} = 73.1 \pm 0.9$ (exp) $\pm 0.6$ (lumi) $\pm 3.8$ (extr.) mb, which is in excellent agreement with the MBR prediction.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>MBR/Experiment</th>
<th>$\sigma_{tot}$ (mb)</th>
<th>$\sigma_{el}$ (mb)</th>
<th>$\sigma_{inel}$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 TeV</td>
<td>MBR</td>
<td>95.4±1.2</td>
<td>28.4±0.3</td>
<td>69.0±1.0</td>
</tr>
<tr>
<td></td>
<td>TOTEM</td>
<td>98.3±0.2±2.8</td>
<td>24.8±0.2±1.2</td>
<td>73.7±3.4</td>
</tr>
<tr>
<td></td>
<td>TOTEM-lum-ind</td>
<td>98.0±2.5</td>
<td>25.2±1.1</td>
<td>72.9±1.5</td>
</tr>
<tr>
<td>8 TeV</td>
<td>MBR</td>
<td>97.1±1.4</td>
<td>27.2±0.4</td>
<td>69.9±1.0</td>
</tr>
<tr>
<td></td>
<td>TOTEM-lum-ind</td>
<td>101.7±2.9</td>
<td>27.1±1.4</td>
<td>74.7±1.7</td>
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<tr>
<td></td>
<td>ATLAS</td>
<td>95.35±1.36</td>
<td>24.00±0.80</td>
<td>71.34±0.90</td>
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<tr>
<td>13 TeV</td>
<td>MBR</td>
<td>103.7±1.9</td>
<td>30.2±0.8</td>
<td>73.5±1.3</td>
</tr>
<tr>
<td></td>
<td>ATLAS</td>
<td>103.7±1.9</td>
<td>30.2±0.8</td>
<td>73.5±1.3</td>
</tr>
</tbody>
</table>

It should be emphasized that the tensor-Pomeron hypothesis predicts directly only the total cross section. As discussed above, the elastic cross section for $1.8 < \sqrt{s} \leq 13$ TeV is obtained from the global fit$^7$, while for $1.8 < \sqrt{s} \leq 13$ TeV we use an extrapolation of the global-fit ratio of $\sigma_{el}/\sigma_{tot}$, which is slowly varying with $\sqrt{s}$, multiplied by $\sigma_{tot}$. The total inelastic cross section is calculated as the difference between the total and elastic. Thus, a measured lower $\sigma_{el}$ would result in a higher $\sigma_{inel}$. As seen in Table 1, the MBR $\sigma_{el}$ is larger than the TOTEM and CMS measurements by $\sim 2$ mb at both $\sqrt{s}=7$ and 8 TeV, which could imply a higher MBR prediction for $\sigma_{inel}$ at 13 TeV by $\sim 2$ mb as well. This interplay between $\sigma_{el}$ and $\sigma_{inel}$ should be kept in mind as measurements of $\sigma_{el}$ and $\sigma_{tot}$ at $\sqrt{s} = 13$ TeV become available in the near future.

4 Summary and conclusions

We have presented predictions of the elastic scattering, total-inelastic, and total proton-proton cross sections at the LHC, based on a Regge theory inspired tensor-Pomeron implementation of the RENORM model for hadronic diffraction. All measured cross sections are in good agreement within the experimental uncertainties of the data and the theoretical uncertainties of the model down to the $\sim 1\%$ level.

Acknowledgments

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References

4. M.G. Albrow, private communication, discussions about the Axial Field Spectrometer results on exclusive $\pi^+\pi^-$. 