Predictions of diffractive cross sections in proton-proton collisions

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Abstract. We review our pre-LHC predictions of the total, elastic, total-inelastic, and diffractive components of proton-proton cross sections at high energies, expressed in the form of unitarized expressions based on a special parton-model approach to diffraction employing inclusive proton parton distribution functions and QCD color factors and compare with recent LHC results.

Keywords: diffraction, pomeron, total cross section

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1. INTRODUCTION

Measurements at the LHC have shown that there are sizable disagreements among Monte Carlo (MC) implementations of “soft” processes based on cross sections proposed by various physics models, and that it is not possible to reliably predict all such processes, or even all aspects of a given process, using a single model [1]. In the CDF studies of diffraction at the Tevatron, all processes are well modeled by the MBR (Minimum Bias Rockefeller) MC simulation, which is a stand-alone simulation based on a unitarized Regge-theory model, RENORM [2], employing inclusive nucleon parton distribution functions (PDF’s) and QCD color factors. The RENORM model was updated in EDS-2009 to include a unique unitarization prescription for predicting the total $pp$ cross section at high energies [3], and has recently been included as an MBR option for simulating diffractive processes in PYTHIA8, as of version PYTHIA8.165 [4], to be referred here-forth as PYTHIA8-MBR. In this paper, we briefly review the cross sections [5] implemented in this option of PYTHIA8 and compare them with LHC measurements.

The PYTHIA8-MBR option includes a full simulation of the hadronization of the implemented diffraction dissociation processes: single, double, and central dissociation. In the original MBR simulation used in CDF, the hadronization of the final state(s) was based on a data-driven phenomenological model of multiplicities and $p_T$ distributions calibrated using $S\bar{p}pS$ and Fermilab fixed-target results. Later, the model was successfully tested against Tevatron MB and diffraction data. However, only $\pi^\pm$ and $\pi^0$ particles were produced in the final state, with multiplicities obeying a statistical model of a modified Gamma distribution that provided good fits to experimental data [6]. This model could not be used to predict specific-particle final states.

In the PYTHIA8-MBR implementation, hadronization is performed by PYTHIA8 tuned to reproduce final-state distributions in agreement with MBR’s, with hadronization done in the PYTHIA8 framework. Thus, all final-state particles are now automatically produced, greatly enhancing the horizon of applicability of this simulation.
2. CROSS SECTIONS

The following diffraction dissociation processes are considered in PYTHIA8-MBR:

- **SD** \( pp \rightarrow Xp \) Single Diffraction (or Single Dissociation), \( (1) \)
- or \( pp \rightarrow pY \) (the other proton survives)
- **DD** \( pp \rightarrow XY \) Double Diffraction (or Double Dissociation), \( (2) \)
- **CD** (or DPE) \( pp \rightarrow pXp \) Central Diffraction (or Double Pomeron Exchange). \( (3) \)

The renormalization predictions are expressed as unitarized Regge-theory formulas, in which the unitarization is achieved by a renormalization scheme where the Pomeron \((P)\) flux is interpreted as the probability for forming a diffractive (non-exponentially suppressed) rapidity gap and thereby its integral over all phase space saturates when it reaches unity. Differential cross sections are expressed in terms of the \(P\)-trajectory, \( \alpha(t) = 1 + \epsilon + \alpha' t = 1.104 + 0.25 \text{ (GeV}^{-2}\text{)} \cdot t \), the \(P\)-\(P\) coupling, \( \beta(t) \), and the ratio of the triple-\(P\) to the \(P\)-\(P\) couplings, \( \kappa \equiv g(t)/\beta(0) \). For large rapidity gaps, \( \Delta y \leq 3 \), for which \(P\)-exchange dominates, the cross sections may be written as,

\[
d^2\sigma_{SD} = \frac{1}{N_{gap}(s)} \left[ \frac{\beta^2(t)}{16\pi} e^{2[\alpha(t)-1]\Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left( \frac{s'}{s} \right)^\epsilon \right\},
\]

\[
d^3\sigma_{DD} = \frac{1}{N_{gap}(s)} \left[ \frac{\kappa \beta^2(0)}{16\pi} e^{2[\alpha(t)-1]\Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left( \frac{s'}{s} \right)^\epsilon \right\},
\]

\[
d^4\sigma_{DPE} = \frac{1}{N_{gap}(s)} \left[ \Pi_i \left[ \frac{\beta^2(t_i)}{16\pi} e^{2[\alpha(t_i)-1]\Delta y_i} \right] \right] \cdot \kappa \left\{ \kappa \beta^2(0) \left( \frac{s'}{s} \right)^\epsilon \right\},
\]

where \( t \) is the 4-momentum-transfer squared at the proton vertex, \( \Delta y \) the rapidity-gap width, and \( y_0 \) the center of the rapidity gap. In Eq. (6), the subscript \( i = 1,2 \) enumerates Pomerons in the DPE event, \( \Delta y = \Delta y_1 + \Delta y_2 \) is the total rapidity-gap (sum of two gaps) in the event, and \( y_c \) is the center in \( \eta \) of the centrally-produced hadronic system.

The total cross section \( (\sigma_{tot}) \) is expressed as:

\[
\sigma^p_{tot} = 16.79s^{0.104} + 60.81s^{-0.32} + 31.68s^{-0.54} \quad \text{for} \ \sqrt{s} \leq 1.8 \text{ TeV},
\]

\[
\sigma^p_{tot} = \sigma_{tot}^{CDF} + \frac{\pi}{s_0} \left[ \left( \ln \frac{s}{s_F} \right)^2 - \left( \ln \frac{\sqrt{s}}{s_F} \right)^2 \right] \quad \text{for} \ \sqrt{s} \geq 1.8 \text{ TeV},
\]

where \( s_0 \) and \( s_F \) are energy and (Pomeron flux) saturation scales, respectively [5]. For \( \sqrt{s} \leq 1.8 \text{ TeV} \), where there are Reggeon contributions, we use the global fit expression [7], while for \( \sqrt{s} \geq 1.8 \text{ TeV} \), where Reggeon contributions are negligible, we employ the Froissart-Martin formula [8, 9, 10]. The two expressions are smoothly matched at \( \sqrt{s} \geq 1.8 \text{ TeV} \).

The elastic cross section is obtained from the global fit [7] for \( \sqrt{s} \leq 1.8 \text{ TeV} \), while for \( 1.8 < \sqrt{s} \leq 50 \text{ TeV} \) we use an extrapolation of the global-fit ratio of \( \sigma_{el}/\sigma_{tot} \), which is slowly varying with \( \sqrt{s} \), multiplied by \( \sigma_{tot} \). The total non-diffractive cross section is then calculated as \( \sigma_{ND} = (\sigma_{tot} - \sigma_{el}) - (2\sigma_{SD} + \sigma_{DD} + \sigma_{CD}) \).
3. RESULTS

In this section, we present as examples of the predictive power of the RENORM model some results reported at this conference by the ALICE and TOTEM collaborations for pp collisions at 7 TeV that can be directly compared with RENORM formulas without the use of the PYTHIA8-MBR Monte Carlo simulation. Figure 1 (left) shows a comparison of the TOTEM total, elastic, and total-inelastic cross sections, along with results from other experiments, fitted by the COMPETE collaboration [11]; the RENORM predictions, entered as filled squares, are in excellent agreement with the TOTEM results. Similarly, in Fig. 1 (right), excellent agreement is observed between the ALICE total-inelastic cross sections at $\sqrt{s} =0.9$, 2.76 and 7 TeV and RENORM (Goulianos) predictions [12].

Another example of the (absolute) predictive power of RENORM is shown in Fig. 2.

![Figure 1](image1.png)

**FIGURE 1.** (left) TOTEM measurements of the total, the total-inelastic, and elastic pp cross sections at $\sqrt{s} = 7$ TeV shown with best COMPETE fits [11]; RENORM predictions were added as filled squares. (right) ALICE measurements [12] of the total inelastic cross section at $\sqrt{s} = 0.9, 2.76$ and 7 TeV compared to theoretical predictions. Excellent agreement with the RENORM (Goulianos) predictions is observed.

![Figure 2](image2.png)

**FIGURE 2.** ALICE measured single- and double-diffractive cross sections compared to various theoretical models, including RENORM (from Ref. [12]; curve definitions and colors same as in Fig. 1). Good agreement between data and RENORM predictions is observed within the experimental uncertainties.
4. SUMMARY

We reviewed our pre-LHC predictions for the total, elastic, total-inelastic, and diffractive components of proton-proton cross sections at high energies, which are based on a special unitarized parton-model approach to diffraction employing inclusive proton parton distribution functions and QCD color factors. We discussed single diffraction, double diffraction and central diffraction or double-Pomeron exchange, and compared predictions of the model with LHC measurements.

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REFERENCES

1. See models presented by various authors in (these) Proceedings of DIFFRACTION 2012.